4.6 - The Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra

Theorem If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

Equation	Roots	Degree of equation Number of roots
$x + 1 = 0$	-1	1
$x^2 - 4x + 13 = 0$	$2 + 3i,\ 2 - 3i$	2
$x^3 + 4x^2 + 4x = 0$	$0, -2, -2$	3
$x^4 - 10x^2 + 9 = 0$	$1, -1, 3, -3$	4
$x^5 - 16x = 0$	$0, 2, -2, 2i, -2i$	5

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Descartes's Rule of Signs

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial function with real coefficients.

- The number of *positive real zeros* of f is equal to the number of changes in sign of the coefficients of $f(x)$ or is less than this by an even number.
- The number of *negative real zeros* of f is equal to the number of changes in sign of the coefficients of $f(-x)$ or is less than this by an even number.

How many positive and negative real zeros does the function have? How many imaginary zeros?

$$f(x) = x^6 + x^5 - 17x^4 - 15x^3 + 14x^2 - 16x + 32$$

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Decartes's Rule of Signs

$$P(x) = \overrightarrow{x^5} - 4x^4 - \overrightarrow{6x^3} + \overrightarrow{6x^2} - \overrightarrow{7x} + 10$$

$$P(-x) = -x^5 - \overrightarrow{4x^4} + 6x^3 + 6x^2 + 7x + 10$$

+ Real Roots	- Real Roots	Img Roots	Total Roots

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Decartes's Rule of Signs

Positive = 5 changes

Negative = 2 changes

Total = degree 9

+ Real Roots	- Real Roots	Img Roots	Total Roots